Eleanor Roosevelt Community Learning Center

Learning the Multiplication Combinations

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-----	1	2	3	4	5	6	7	8	9	10	11	12
2	-----	-----	4	6	8	10	12	14	16	18	20	22	24
3	-----	-----	-----	9	12	15	18	21	24	27	30	33	36
4	-----	-----	-----	-----	16	20	24	28	32	36	40	44	48
5	-----	-----	-----	-----	-----	25	30	35	40	45	50	55	60
6	-----	-----	-----	-----	-----	-----	36	42	48	54	60	66	72
7	-----	-----	-----	-----	-----	-----	-----	49	56	63	70	77	84
8	-----	-----	-----	-----	-----	-----	-----	-----	64	72	80	88	96
9	-----	-----	-----	-----	-----	-----	-----	-----	-----	81	90	99	108
10	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	100	110	120
11	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	121	132
12	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	144

The colors in this table indicate groups of combinations that can be learned together.

(A printout of this table can also be used as a graphical checklist of the combinations that have been mastered and a reminder of how few remain to be learned.)

Resources to print:

- 100's Squares (Numbered), 100's Squares (Blank)
- Flash Cards: Front, Flash Cards: Back
- Flashcard Drill Progress Chart

Before launching into learning/teaching the multiplication combinations, several general "philosophical" points are in order:

- Given the widespread availability of calculators, understanding multiplication is more important than knowing the multiplication table. You need to know a multiplication problem when you see one, or you won't be able to do it even using a calculator.
- That said, it is still important to know the multiplication tables at least up to 10 x 10.
  - Mental computation, at least for small numbers, and paper-and-pencil computation, at least for moderate sized numbers, is still an important life skill.
  - Reducing fractions requires factoring. For example to reduce 12/32 you need to recognize that 4 divides into both 12 and 32. A pocket calculator isn't very useful in this process.
  - The ability to factor numbers is also important in algebra.
- Once you have learned the multiplication table up through the 10's, the 11's are almost automatic because of the simplicity of the pattern. The primary rationale for pushing on to the 12's is the

because of the simplicity of the pattern. The primary rationale for pushing on to the 12's is the frequency of 12's in informal calculations (converting feet to inches, packaging by dozens, 24 hours per day (two 12 hour cycles), etc.).

- A young student who is learning the multiplication table for the first time (and not running into major problems) can take advantage of the mindset to go all the way to the 12's. However, in the case of an older student who has experienced repeated failure, it is reasonable to stop at the 10's.

- The multiplication table should first be learned in terms of patterns so the combinations make sense in some way. (Different patterns will "stick" for different people.) But ultimately, the combinations should become "automatic". Before "declaring victory" a student should be able to go through a set of flash cards without hessitation.

- A "100's Board" can be very helpful in revealing patterns in the multiplications combinations. Use blank 100's board sheets and have the student "skip-count" by all the numbers from 1 through 12.

  - The 2's and 5's fall into vertical columns. (Notice that 2 and 5 both divide evenly into 10.)

1	**2**	3	**4**	5	**6**	7	**8**	9	**10**
11	**12**	13	**14**	15	**16**	17	**18**	19	**20**
21	**22**	23	**24**	25	**26**	27	**28**	29	**30**
31	**32**	33	**34**	35	**36**	37	**38**	39	**40**
41	**42**	43	**44**	45	**46**	47	**48**	49	**50**
51	**52**	53	**54**	55	**56**	57	**58**	59	**60**
61	**62**	63	**64**	65	**66**	67	**68**	69	**70**
71	**72**	73	**74**	75	**76**	77	**78**	79	**80**
81	**82**	83	**84**	85	**86**	87	**88**	89	**90**
91	**92**	93	**94**	95	**96**	97	**98**	99	**100**

1	2	3	4	**5**	6	7	8	9	**10**
11	12	13	14	**15**	16	17	18	19	**20**
21	22	23	24	**25**	26	27	28	29	**30**
31	32	33	34	**35**	36	37	38	39	**40**
41	42	43	44	**45**	46	47	48	49	**50**
51	52	53	54	**55**	56	57	58	59	**60**
61	62	63	64	**65**	66	67	68	69	**70**
71	72	73	74	**75**	76	77	78	79	**80**
81	82	83	84	**85**	86	87	88	89	**90**
91	92	93	94	**95**	96	97	98	99	**100**

  - The 3's and 9's form diagonal patterns. Note that 9 falls 1 short of completing each row, which sets it back by one in the next row.

1	2	**3**	4	5	**6**	7	8	**9**	10
11	**12**	13	14	**15**	16	17	**18**	19	20
21	22	23	**24**	25	26	**27**	28	29	**30**
31	32	**33**	34	35	**36**	37	38	**39**	40
41	**42**	43	44	**45**	46	47	**48**	49	50
51	52	53	**54**	55	56	**57**	58	59	**60**
61	62	**63**	64	65	**66**	67	68	**69**	70
71	**72**	73	74	**75**	76	77	**78**	79	80
81	82	83	**84**	85	86	**87**	88	89	**90**
91	92	**93**	94	95	**96**	97	98	**99**	100

1	2	3	4	5	6	7	8	**9**	10
11	12	13	14	15	16	17	**18**	19	20
21	22	23	24	25	26	**27**	28	29	30
31	32	33	34	35	**36**	37	38	39	40
41	42	43	44	**45**	46	47	48	49	50
51	52	53	**54**	55	56	57	58	59	60
61	62	**63**	64	65	66	67	68	69	70
71	**72**	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	**90**
91	92	93	94	95	96	97	98	**99**	100

  - 4's will result in a staggered pattern. For 2-digit numbers, when the first digit is even, the last digit is 4, 8, or 0. When the first digit is odd, the last digit is 2 or 6.

1	2	3	**4**	5	6	7	**8**	9	10
11	**12**	13	14	15	**16**	17	18	19	**20**
21	22	23	**24**	25	26	27	**28**	29	30
31	**32**	33	34	35	**36**	37	38	39	**40**
41	42	43	**44**	45	46	47	**48**	49	50
51	**52**	53	54	55	**56**	57	58	59	**60**
61	62	63	**64**	65	66	67	**68**	69	70
71	**72**	73	74	75	**76**	77	78	79	**80**
81	82	83	**84**	85	86	87	**88**	89	90
91	**92**	93	94	95	**96**	97	98	99	**100**

- Once the patterns have been recognized and discussed, flash cards are one of the most efficient ways to "drill". (Printed flashcards are also available at ERCLC. Also, a set of flash cards is provided here as two Adobe Acrobat (pdf) files. The first file contains the problem statements and the second file contains the answers. Print the problems from the first file, then print the answers on the back of the problems. Alternatively print out one set of each and use it as a duplication master. Print on heavy opaque stock and check to make sure the answers align with the correct problem statements.)

- The best proceedure for using flash cards is:
  - go through the stack once, putting the cards into "correct" and "error" piles.
  - list the errors and review them.
  - mark the number missed on the first pass on a cumulative graph to follow one's progress
  - after reviewing, go through the missed cards (or missed and slow cards) repeatedly until none are missed.
- Keep a graph of the number missed. Keeping a graph makes progress more tangible. As you make progress the height of the graph shows that the task remaining is shrinking.
  - Continue daily flashcard drills until the number of errors goes to zero reliably (several days in a row).
  - Then don't drill for a week.
  - Start drilling again until the errors go back to zero reliably.
  - Don't drill for another week.
  - Start drilling again until the errors go back to zero.
  - The reason for the week-long breaks is to make sure the memorized material has moved into long-term memory. Both the breaks and the resumed drilling are important to the process.

Notes on the groupings

Most of the combinations are extremely easy to learn. These are the ones indicated in light blue.

- 0 times any number is 0.
- 1 times any number is simply that number.
- (It is important to check for confusion between the 0's rule and the 1's rule, especially in the case of 0 x 1.)
- 2 times any number is simply the number added to itself.
- 10 times any whole number is found by simply adding a zero. (This rule gets modified later to become "move the decimal one place to the right" when working with decimal numbers.)
- 11 times any number up to 9 is simply the digit written twice.

The combinations marked in light green can be learned by an easy rule.

- The 11's rule for 2-digit numbers (whose sum is less than 10) is to split the two digits and put their sum in the middle. Thus in the case of 11 x 12, write 1__2 and then add 1 and 2 to get 3 and put this in the middle: 132. Thus 11 x 34 = 374.

- 5 times any even number is simply half of 10 times that number. Since 10 x 8 = 80, 5 x 8 = 40. Since 10 x 12 = 120, 5 x 12 = 60. (One might also remember 5 x 12 by remembering that there are 5 minutes between the 12 numbers on a clock and there are 60 minutes in an hour.)
- 5 times any odd number ends in a 5 and fits between the results for the even numbers. Thus: 5 x 6 = 30, 5 x 8 = 40, 5 x 10 = 50, so 5 x 7 = 35 and 5 x 9 = 45.
- 9 times any number will result in a number whose digits add up to 9. (This is true even for large numbers if the digits are added up repeatedly until they come out to a single digit: e.g. the digits of 2187 add up to 18 and the digits of 18 add up to 9.) 9 times any single digit number starts with 1 less than the number and ends with whatever makes it add up to 9. Thus 9 x 5 starts with 4 and since 4 + 5 = 9, the answer is 45.

The 3's and 4's require outright memorization, but the numbers are fairly small and there are patterns that can help.

- In the 3's, the products come in groups of 3:
  - 3 times 1, 2, and 3 equal 3, 6, and 9 (Single digits)
  - 3 times 4, 5, and 6 equal 12, 15, and 18 (Teens)
  - 3 times 7, 8, and 9 equal 21, 24, and 27 (Twentys)
  - Note that in each group the first digit increases by 1 and the last digit decreases by 1, so the sum of the digits remains the same.
  - In ***every*** multiple of 3, the sum of the digits is either 3, 6, or 9 (except for 3 x 0 = 0).

- In the 4's, the last digits are staggered:
  - 4 times 0, 1, and 2 equal 0, 4 and 8.
  - 4 times 3 and 4 equal 12, and 16.
  - 4 times 5, 6, and 7 equal 20, 24, and 28.
  - 4 times 8 and 9 equal 32 and 36.
  - Note that for each block of products with the same first digit the last digits alternate between [0, 4, 8] and [2, 6]
  - If the first digit is even, the last digit is 0, 4, or 8.
  - If the first digit is odd, the last digit is 2 or 6.

Each set of numbers really needs only to be learned from the number times itself on up. That is because if one of the numbers is smaller, it will have been learned earlier.

- Once you know the 3's, the results for the 6's are twice as great:
  - 3 x 6 = 18, so 6 x 6 = 36
  - 3 x 7 = 21, so 6 x 7 = 42
  - 3 x 8 = 24, so 6 x 8 = 48
  - 3 x 12 = 36, so 6 x 12 = 72
- Once you know the 6's, the results for the 12's are twice as great:
  - 6 x 6 = 36, so 6 x 12 = 72
  - 6 x 7 = 42, so 7 x 12 = 84
  - 6 x 8 = 48, so 8 x 12 = 96
  - 6 x 9 = 54, so 9 x 12 = 108

That leaves just 3 combinations to learn outright:

- 7 x 7 = 49 (just remember it!)
- 7 x 8 = 56 (notice that the digits of the problem and the answer are 5, 6, 7, and 8!)
- 8 x 8 = 64 (can be seen as the double of 4 x 8 which is 32)

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